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THE THEORY OF ISOSTASY

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SECTION I. INTRODUCTION

GENERAL

According to the present general conception the theory of isostasy consists of two main postulates, first that the elevated portions of the earth are deficient in density, and second that the material of the earth is comparatively weak. It is generally accepted that these two postulates are inseparable, for it is argued on the one hand that, if the elevated portions are deficient in density, readjustment involving deformation and failure must have taken place in order to compensate for the large mass of material eroded from the lands and deposited in the sea; and it is contended on the other hand that, if the earth is weak, it could not support the mountains and continents unless they are compensated by a defect of density below. In accordance with these two main postulates it is conceived that the dominant type of earth deformation consists in vertical movements between various segments accompanied by lateral flowage of rock beneath and possibly crumpling of the rock in the border zones. This conception is not only applied to the major earth segments, the continents and oceans, but to the smaller units of the continents as well.

The theory of isostasy is a decided contrast to the alternative conception that the earth is strong enough to support the continents and mountains even though there are no compensating density differences, that changes of weight at the surface do not produce vertical movements of the segments in a weaker substratum, and that the dominant type of deformation is folding and upwarping due to lateral compression.

The theory of isostasy if correct would be of fundamental importance to the geologist in interpreting earth movements.

Previous to Hayford's geodetic investigation the conceptions of isostasy were largely speculative. After a comprehensive study of the deflections of the plumb bob carried out by the United States Coast and Geodetic Survey under Hayford's direction,[1] the rather startling conclusions were reached that the excesses of mass composing the continents and mountains are completely compensated by deficiency of density below and that this deficiency of density extends to a depth of something like 60 to 150 miles. These conclusions lent strong support to the theory of isostasy. Considering the completeness of the density compensation, there seemed to be no escape from the conclusion that readjustments of the nature postulated by isostasy are continually taking place.

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[1] The most complete statement of Hayford's work which has been published is contained in his two reports issued in 1909 and 1910 by the U.S.C. and G.S. and entitled, *The Figure of the Earth and Isostasy from Measurements in the United States* and *Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy.*

With the theory of isostasy apparently on such a firm basis, Hayford and others have elaborated the conceptions of earth movements involved in the theory, but these further inferences have not met the approval of many geologists.

The possibility that Hayford had made an error in his geodetic work suggested an investigation which is the basis of this paper. The attempt has been made, first, to examine Hayford's geodetic work apart from any inferences which may have been drawn from it, and second, to examine the theory of isostasy with reference to inferences from geodetic evidence and also on general grounds. This paper has accordingly been divided into two main parts entitled "The Geodetic Work of John F. Hayford" and "The Theory of Isostasy." A "Summary of Conclusions" is given at the end.

It seemed highly desirable in connection with the criticism of Hayford's geodetic work that several of the terms employed should be defined.

DEFINITIONS

"*'Isostatic compensation' is the compensation of the excess of matter at the surface (continents) by the defect of density below, and of the surface defect of matter (oceans) by excess of density below.*"[1] "Isostatic compensation" will also be referred to simply as "compensation" and an area or segment of the earth will be spoken of as "compensated" if there is isostatic compensation of the excess or defect of matter over that area or at the surface of the given segment. From the above definition it follows that there will be, in general, a density difference between an average sea level segment

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[1] *The Figure of the Earth and Isostasy*, U.S.C. and G.S., 1909, p. 67.

of the earth and a compensated segment, the surface of which is not at sea level. *This density difference will be called the "compensating density difference."*

The "depth of compensation" for any segment of the earth is the greatest depth below sea level at which there is a compensating density difference. This is different from the definition of Hayford which makes the "depth of compensation" the depth "within which the isostatic compensation is complete." The former definition allows for the possibility of a compensation which is not complete.

The "distribution of compensation" for any segment of the earth is the manner of variation of the compensating density difference with respect to depth. If the compensating density difference is uniform, the distribution of compensation is uniform; if it is uniformly varying from a maximum at the surface to zero at the depth of compensation the distribution of compensation is uniformly varying.

The "degree of completeness of isostatic compensation" is an expression used by Hayford. After defining the depth of compensation as quoted above, he says, "At and below this depth the condition as to stress of any element of mass is isostatic; that is, any element of mass is subject to equal pressures from all directions as if it were a portion of a perfect fluid. . . . . In terms of masses, densities, and volumes, the condition above the depth of compensation may be expressed as follows: The mass in any prismatic column which has for its base a unit area of the horizontal surface which lies at the depth of compensation, for its edges vertical lines (lines of gravity) and for its upper limit the actual irregular surface of the earth (or sea surface if the area in question is beneath the ocean) is the same as the mass in any other similar prismatic column having any other unit area of the same surface for its base." This condition of course follows from Hayford's definition of depth of compensation, but it would not hold for the definition adopted in this discussion unless the compensation were complete. Hayford continues as follows: "If this condition of equal pressures, that is, of equal superimposed masses, is fully satisfied at a given depth the compensation is said to be complete at that depth. If there is a variation from equality of superimposed masses the differences may be taken as a measure of the degree of incompleteness of compensation."[1] In order to make this idea exact, let A and B (Fig. 1) represent two columns each of horizontal cross-section, a, and extending to the depth of compensation, h_1, the upper surface of A being h miles above sea level and the upper surface of B being at sea level. Let the weight of A equal W_A and the weight of B equal W_B. If there were no isostatic compensation, and if the densities of A and B were the same at similar depths, then W_A would be in excess of W_B by the amount $a\delta h$, where δ is the mean surface density of the earth. If this excess of weight were entirely made up for by a deficiency of density below, compensation would be complete. *Therefore let the "degree of completeness of isostatic compensation" for any segment, A, be defined as*

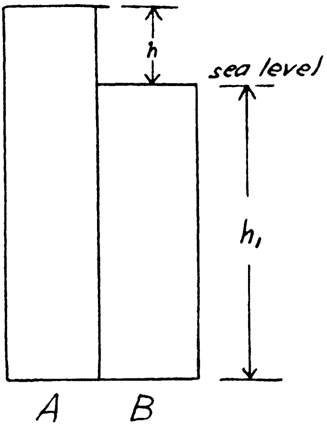

FIG. 1

$$M=\frac{a\delta h-(W_A-W_B)}{a\delta h}$$

The quantity in parenthesis is the amount by which the weight of A is in excess of the weight of B. The whole numerator is, therefore, the weight which has been made up for by a deficiency of density below the surface and the entire fraction is a number expressing the part of the weight, $a\delta h$, which has been made up for. The above formula holds equally well for land and ocean areas if h be taken positive above sea level and negative below.

FOR LAND AREAS

1. If $W_A < W_B$ then $M > 1$
2. If $W_A = W_B$ then $M = 1$
3. If $W_A - W_B = a\delta h$ then $M = 0$
4. If $W_A - W_B > a\delta h$ then $M < 0$

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[1] *The Figure of the Earth and Isostasy*, 1909, p. 67.

FOR OCEAN AREAS (h BEING A NEGATIVE QUANTITY)

1. If $W_A > W_B$ then $M > 1$
2. If $W_A = W_B$ then $M = 1$
3. If $W_A - W_B = a\delta h$ then $M = 0$
4. If $W_A - W_B < a\delta h$ then $M < 0$

When $M = 0$, there would be no isostatic compensation. The condition that M is negative is equivalent to a distribution of density so related to the surface that the material under any surface is heavier than the material under a lower surface. There would be no isostatic compensation in this case. In view of the facts brought out by Hayford the fourth case is, however, very improbable.

"Over-compensation" is such an isostatic compensation that $M > 1$.

"Complete compensation" is such an isostatic compensation that $M = 1$.

"Under-compensation" is such an isostatic compensation that $0 < M < 1$.

Isostatic compensation is considered the more complete, the closer M approaches to 1.

SECTION II. THE GEODETIC WORK OF JOHN F. HAYFORD

BRIEF DESCRIPTION OF HIS WORK AND METHODS

On certain assumptions as to the size and shape of the earth and as to the position of a base station on this ideal earth, Hayford, by triangulation and geodetic observations, measured the prime vertical and meridian components of the deflection of the plumb bob from the true vertical at several hundred stations scattered over the United States. He then calculated the deflections which all the topographic features within a radius of 2,564 miles of each station should produce if the density of the earth were the same at similar depths. He found that these calculated deflections, which he called the "topographic deflections," were universally larger than the "observed deflections." The only explanation of such widespread observations is that there is some sort of isostatic compensation of the surface excesses and defects of mass. Recognizing this fact Hayford set out to make a series of least square solutions assuming various kinds of isostatic compensation. He calculated what the deflections should be assuming isostatic compensation by the use of a reduction factor, that is, a factor which when multiplied by the topographic deflection will give the deflection, isostatic compensation considered. In all of his solutions Hayford assumed the isostatic compensation to be complete. In his five principle solutions he assumed a uniform distribution of compensation and assumed depths of compensation varying from zero to infinity.[1] On these assumptions the conclusion was reached that the most probable depth of compensation is 76 miles since the sum of the squares of the residuals was least for this depth.

Subsidiary solutions were made assuming (1) that the compensation is uniformly distributed in a ten-mile substratum, (2) that the compensation is greatest at the surface and decreases uniformly with respect to depth until it becomes zero at the depth of compensation, and (3) that the compensation is distributed according to the law postulated by Chamberlin.[2] The method used in each of these three cases was to find the depth for which the reduction factor was most like the reduction factor for the most probable solution assuming a uniform distribution. Hayford concluded that so far as the geodetic evidence available could test them, any of the three distributions of compensation postulated is as probable as a uniform distribution. The depth of compensation found for a distribution in a ten-mile substratum was 40 miles; for a uniformly varying distribution, 117 miles; and for the Chamberlin distribution of compensation, 193 miles.

A further interesting phase of Hayford's work is his C-solution[3] which was made on the assumption that there is no isostatic compensation under land areas but that there is complete isostatic compensation at depth zero under ocean areas. It was found that these assumptions were not as close to the facts as the assumption of complete compensation at the depth of 76 miles under both land and ocean. In discussing the C-solution Hayford says:[1]

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[1] The condition that the depth of compensation is infinite is taken as equivalent to no isostatic compensation. The condition that the depth of compensation is zero is taken as equivalent to the condition that the topographic features do not affect the plumb bob.

[2] This law postulates a maximum density difference slightly below the surface. This density difference decreases rapidly at first and then more gradually with respect to depth.

[3] P. 168 of 1909 report.

> It follows, moreover, that it is an isostatic compensation of the separate topographic features of the continent, not a compensation merely of the continent as a whole. In solution A[2] it is a compensation of the separate features which is assumed. An inspection of the numerical values of the computed topographic deflections, and of the deflections computed with isostatic compensation considered shows that merely to have assumed the continent as a whole to be compensated, not its separate topographic features would have given a solution resembling solution C much more closely than solution A.[3]

A very vital step in Hayford's work is his determination of the degree of completeness of compensation. As this is one of the principal points to be criticized, his method will be explained in detail in connection with the criticism. Suffice it to say here that he concluded that the isostatic compensation is on an average nine-tenths complete.

CRITICISM OF HAYFORD'S WORK

Hayford certainly showed that there is some sort of isostatic compensation; but he did not fully consider all possibilities as to the nature of this compensation. The exact nature of isostatic compensation for any place is determined by three factors, (1) depth of compensation, (2) distribution of compensation, and (3) degree of completeness of compensation. Hayford considered all possible depths of compensation and several distributions of compensation; but all of his solutions involving isostatic compensation were made on the assumption that the compensation is complete. This was a purely arbitrary assumption on Hayford's part since he gave no reason whatever for believing at the outset that compensation is complete, and furthermore the fact that he later attempted to find the degree of completeness implies that there is no reason to believe at the outset in complete compensation. In view of this fact the method of determining the degree of completeness of compensation is questionable.

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[1] Quoted from p. 169 of 1909 report.

[2] Solution *A* was made on the assumption that compensation is complete at depth zero over both land and sea. This solution turned out to be nearer the truth than solution C.

[3] It should be noted that the assumption of complete compensation under ocean areas with no compensation under continents is not equivalent to a compensation of the continents as a whole with respect to the oceans, but only to a compensation for the part of the continents below sea level.

Criticism of method of finding degree of completeness of compensation.—Hayford's method is best explained by an example. Suppose the topographic deflection at some station is 35.79″. This is assuming no isostatic compensation. Suppose that the residual assuming complete compensation at a depth of 76 miles is 3.33″, or in other words, suppose that the difference of the true deflection and the deflection which would exist if the compensation were complete at a depth of 76 miles is 3.33″. This value (3.33″) is apparently the part of the topographic deflection which has not been made up for by isostatic compensation. The ratio of 3.33 to 35.79 is, therefore, taken by Hayford as a measure of the incompleteness of compensation. In explanation of this method Hayford writes as follows:

> The residuals of solution G[1] furnish a test of the departures of the facts from the assumed condition of complete isostatic compensation uniformly distributed to a limiting depth of 113.7 kilometers. In order to obtain definite ideas let the whole of the residuals of this solution be credited to the incompleteness of the compensation. The conclusion as to the completeness of compensation will then be in error in that the actual approach to completeness will be considerably closer than that represented by the conclusion—that is, the conclusion will be an extreme limit of incompleteness rather than a direct measure. For by this process of reasoning every portion of a residual of solution G, due to the departure of the actual distribution of compensation with respect to depth from the assumed distribution, or due to the error in the assumed mean depth of compensation, or to regional variation from a fixed depth of compensation, or due to errors of observation in the astronomic determinations and the triangulation which affect the observed deflection of the vertical, or due to errors of computation, is credited to incompleteness of compensation.[2]

The objection to the paragraph quoted is that it apparently is taken for granted that the error in the assumed mean depth of compensation increased the size of the residuals. Is it not very probable that the introduction of an error in depth actually diminished the residuals? The most probable depth was calculated on the assumption of completeness. If the assumption of completeness was wrong, the depth of compensation which would appear most probable would not be the true depth of compensation but a depth which would counteract the effect of the wrong assumption in regard to completeness. In other words the error in the assumed mean depth of compensation would be such as to decrease the residuals. Therefore the residuals which would have been obtained had the correct depth been used would be larger than the residuals actually obtained. The degree of incompleteness as measured by Hayford's method would, therefore, be larger.

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[1] Solution G, the most probable solution according to the first report, was made assuming a depth of compensation of 70 miles.

[2] Quoted from p. 164 of 1909 report.

If the depth of compensation were known independently, then Hayford's method of finding the completeness would be legitimate. To go back to the example cited before, suppose that it is known independently that the depth of compensation is 25 miles and suppose that the residual obtained on this basis and assuming complete compensation is 15″: this value would be the part of the topographic deflection which had not been made up for by compensation and therefore the ratio of 15 to 35.79 would be an approximate measure of the incompleteness of compensation.

The above argument will be made clear by a brief summary. The depth and degree of completeness of compensation are unknowns to be determined. It is claimed that these two unknowns can not be determined by Hayford's method of assuming complete compensation, calculating the most probable depth, and using the residuals to tell the degree of incompleteness, because this method would only be legitimate for the one case when compensation is actually complete. If compensation were not complete, then Hayford's calculated depth would be wrong and would furthermore be in error in such a direction as to at least partially make up for the wrong assumption regarding degree of completeness. The resulting residuals would therefore not furnish a maximum measure of the degree of incompleteness; but the compensation would appear to be more nearly complete than would be the fact.

We are forced to conclude that, from the geodetic evidence alone, neither the depth nor the degree of completeness of isostatic compensation can as yet be considered settled.[1]

Criticism of C-solution.—The criticism might be made of the C-solution that the assumption of complete compensation at depth zero under oceans obviously does not correspond to the facts and that, by trying several depths, a combination might be found which would appear to be, so far as the information available could test it, as close to the actual conditions as any other hypothesis. From the wide departure of the C-solution it seems, however, rather improbable that such a depth could be found.

FURTHER CONSIDERATIONS

Possibilities of an incomplete compensation.—Since Hayford only considered the case of complete compensation, it is desirable to see whether or not an incomplete compensation would meet the geodetic requirements as well as a complete compensation. Any test of incomplete compensation based on Hayford's residuals is apt to be misleading since these residuals may involve two errors that tend to counterbalance each other. By a study of the reduction factor, however, we may be able to tell whether or not an incomplete compensation would be as probable from the geodetic point of view as complete compensation.

According to the definition given in this paper the degree of completeness of compensation is

$$M=\frac{a\delta h-(W_A-W_B)}{a\delta h}\,. \tag{1}$$

If there is isostatic compensation, there will be a compensating density difference between the material in column A and the material in column B.[2] If at any given depth the density of column A be δ_A and of column B, δ_B, then the compensating density difference at that depth will be $\delta_1=\delta_A-\delta_B$ which is of course negative when A is a land segment. It follows that $\delta_B=\delta_A-\delta_1$. Now

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[1] It should be noted that so far as the nature of compensation is questionable, Hayford's values for the size and figure of the earth are also open to question.

[2] See definition of M, p. 607–8.

$W_A - a\int_0^{(h_1+h)} \delta_1 dz$ is the weight[1] which the column A would have if its density were the same as in column B and is therefore equal to the weight of B plus the weight which the material in A above sea level would have if there were no isostatic compensation. Thus

$$W_A - a\int_0^{(h_1+h)} \delta_1 dz = W_B + a\delta h$$

or

$$W_A - W_B = a\delta h + a\int_0^{(h_1+h)} \delta_1 dz \tag{2}$$

Substituting (2) in (1),

$$M = \frac{a\delta h - \left\{ a\delta h + a\int_0^{(h_1+h)} \delta_1 dz \right\}}{a\delta h}$$

or

$$\delta h M = -\int_0^{(h_1+h)} \delta_1 dz \tag{3}$$

The case of a uniformly distributed compensation will be considered. In this case, δ_1 being a constant, (3) reduces to

$$\delta h M = -\delta_1 (h_1 + h) \tag{4}$$

As stated before Hayford only considers the case where $M=1$. He further makes the approximation of neglecting h in comparison with h_1. This approximation which is permissible for depths of compensation considered by Hayford but which would not be allowable for shallow depths is discussed later. The relation corresponding to (4) which Hayford uses is $\delta h = -\delta_1 h_1$. If however the unknown quantity, M, is retained, the corresponding reduction factor which we will call F_M is as follows:

$$F_M = 1 - M\frac{\log \dfrac{r^1 + \sqrt{(r^1)^2 + h_1^2}}{r_1 + \sqrt{r_1^2 + h_1^2}}}{\log \dfrac{r^1}{r_1}}. \tag{5}$$

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[1] As δ_1 may vary with the depth it is necessary to sum up the product of δ_1 and the small elements of depth rather than use the product, $\delta_1(h_1+h)$.

Adding and substracting M, we have

$$F_M = 1 - M(1-F) \tag{6}$$

where F, the reduction factor obtained by Hayford, is

$$F = 1 - \frac{\log \dfrac{r^1 + \sqrt{(r^1)^2 + h_1^2}}{r_1 + \sqrt{r_1^2 + h_1^2}}}{\log \dfrac{r^1}{r_1}}\,. \tag{7}$$

Comparing F with F_M we see that, when $O<M<1$, F_M will be greater than F for the same ring[1] and h_1. Also for any given ring F is larger, the larger the depth of compensation,[2] h_1. It follows, therefore, that F_M for $M<1$ calculated on any given h_1 will be greater for all rings than the corresponding factor, F, calculated on a smaller h_1, for

$$\Big[F_M\Big]_{\substack{M<1\\ h_1=x}} > \Big[F\Big]_{h_1=x} > \Big[F\Big]_{h_1<x}$$

or

$$\Big[F_M\Big]_{\substack{M<1\\ h_1>76}} > \Big[F\Big]_{h_1=76}$$

Now assuming $M=1$ Hayford has already shown that the set of factors obtained when $h_1=76$ miles gives a closer result than the set of larger factors obtained when $h_1>76$ miles. It seems probable therefore that, if a solution were to be attempted assuming $O<M<1$, nothing would be gained in taking a depth of compensation larger than the most probable depth assuming $M=1$. The writer would not care to make the preceding statement as a positive fact without an inspection of the data for the calculation of the topographic deflections. For it seems possible, although not probable, that a combination of $M<1$ and $h_1>76$ might yield as close a result as $M=1$ and $h_1=76$ on account of the fact that, though the reduction factor becomes larger, the relative increase in the various rings is not the same as the increase when h_1 is made larger than 76 but M is kept equal to 1.

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[1] In calculating the topographic deflection the area around any station is divided into concentric rings whose outer radii are r^1 and inner radii, r_1.

[2] See table, p. 70 of 1909 report.

On the other hand, when $0 < M < 1$ and $h_1 < 76$, the resulting factor, for any given ring, compared to F for $h_1 = 76$, tends to become larger on account of taking $M < 1$, but tends to become smaller on account of taking $h_1 < 76$. If h_1 be taken sufficiently small, the factor, F_M for $M < 1$, and $h_1 < 76$, becomes smaller for certain rings and larger for other rings than F for $h_1 = 76$. It therefore seems quite probable that a combination of $M < 1$ and $h_1 < 76$ should prove to be as close an approximation to the facts as $M = 1$ and $h_1 = 76$.

Similarly, if the case where $M > 1$ were to be considered, the best depth of compensation would probably turn out to be greater than 76 miles.

By equation (6) above, it is an easy matter to calculate the factor F_M for any value of h_1 which is equal to the radius of any ring. In order to obtain a typical example, the factors were calculated assuming $M = .5$ and $h_1 = 19.29$ kilometers (11.987 miles) and are given below together with the factors for Hayford's most probable solution in which it was assumed that $M = 1$ and $h_1 = 113.7$ kilometers (70.67 miles).[1]

It will be noted that, for outer rings, the factor, F_M, is approximately .5 while F is nearly zero. From the examples of calculations of "topographic deflections" given by Hayford it would seem that the outer rings, especially the oceanic compartments, have a considerable effect on the topographic deflection. It might seem, therefore, that $M = .5$, $h_1 = 19.29$ kilometers would not give as close a result as $M = 1$, $h_1 = 113.7$ kilometers; but without some sort of test this question would remain a matter of conjecture. Furthermore the additional hypothesis might be introduced that the compensation under ocean areas is complete or even that ocean areas are over-compensated. Then when we remember that the assumption regarding either M or h_1 or both may be varied, there seems to be nothing to show that a combination of a decided under-compensation and shallow depth under land areas with a practically complete compensation under ocean areas would not prove, so far as present information is able to test it, as close an approximation to the actual conditions as a combination of complete compensation with a depth of 76 miles under both land and sea. And it is not improbable that several combinations involving a decidedly incomplete compensation under land areas could be found which would appear equally as close to the truth as $M=1$ and $h_1=76$ miles.

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[1] The value of the depth of compensation given in the first report as most probable is 70 miles. In the second report 76 miles is given, but the reduction factors for this depth are not published.

Ring	$[F_M]^{M=.5}_{h_1=19.29 \text{ kil.}}$	$[F]_{h_1=113.7 \text{ kil.}}$
29	.996	....
28	.994	....
27	.991	.997
26	.988	.996
25	.982	.995
24	.975	.992
23	.965	.988
22	.950	.983
21	.930	.976
20	.900	.965
19	.860	.951
18	.809	.930
17	.747	.900
16	.679	.859
15	.617	.801
14	.570	.721
13	.539	.618
12	.520	.493
11	.510	.358
10	.505	.234
9	.502	.139
8	.501	.077
7	.500	.040
6	.500	.020
5	.500	.010
4	.500	.005
3	.500	.003
2	.500	.001
1	.500	.001

In the case of $M>1$, $h_1>76$ miles it is probably also true that the compensation under ocean areas would have to be taken complete.

In the foregoing discussion the distribution of compensation was assumed uniform. The argument that, if $M<1$, h_1 is probably less than the most probable depth assuming $M=1$ would hold for the Chamberlin compensation or for a compensation uniformly varying from a maximum at the surface to zero at the depth of compensation. For the reduction factors in the subsidiary investigations are linear functions of the reduction factor for uniform distribution.

Changes in formulae required by shallow depth of compensation.—Under this head the approximation mentioned above in connection with equation (4) will be considered.

$$\delta h M = -\delta_1 (h_1 + h) \tag{4}$$

δ_1 is such a quantity that $(W_A - a\delta_1\ [h_1+h]) = [W_A]_{M=0}$ is the weight which column A would have if there were no isostatic compensation. Therefore

$$W_A = [W_A]_{M=0} + a\delta_1(h_1+h)\ .$$

Thus the deflection due to W_A may be obtained by adding to the deflection which would be produced if there were no isostatic compensation the deflection which would be produced by $a\delta_1(h_1+h)$. Therefore the deflection at any station assuming isostatic compensation is equal to the topographic deflection (D) plus the deflection (D_c) due to the defect or excess of density from the surface down to the depth of compensation. If H be the height of the observing station above sea level, then the deflection due to the defect of density in a compartment whose surface is h miles above sea level is (neglecting the curvature of the earth),[1]

$$D_c = 12.44'' \frac{\delta_1}{\Delta}(\sin a^1 - \sin a_1) \left\{ (H+h_1) \log \frac{r^1 + \sqrt{(r^1)^2 + (H+h_1)^2}}{r_1 + \sqrt{r_1 + (H+h_1)^2}} \right.$$

$$\left. + (h-H) \log \frac{r^1 + \sqrt{(r^1)^2 + (h-H)^2}}{r_1 + \sqrt{r_1^2 + (h-H)^2}} \right\} \tag{8}$$

In calculating the topographic deflection Hayford neglects H except when it is necessary to make a slope correction. However, H is introduced in equation (8) in order to make it exact. Whether it would be legitimate to neglect this factor or not can best be told from the resulting expression for the reduction factor. Taking the value of D_c given in (8) the reduction factor is

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[1] See A. R. Clarke, *Geodesy*, 1880, p. 295, and *Figure of the Earth and Isostasy*, 1909, p. 20, for the derivation of equation (8).

$$F_M=\frac{D+D_c}{D}=1+\frac{1}{\log\frac{r^1}{r_1}}\left\{\frac{\delta_1(H+h_1)}{\delta h}\log\frac{r^1+\sqrt{(r^1)^2+(H+h_1)^2}}{r_1+\sqrt{r_1^2+(H-h_1)^2}}\right. \tag{9}$$

$$\left.+\frac{\delta_1(h-H)}{\delta h}\log\frac{r^1+\sqrt{(r^1)^2+(h-H)^2}}{r_1+\sqrt{r_1^2+(h-H)^2}}\right\}$$

From (4)

$$\frac{\delta_1}{\delta}=-\frac{Mh}{h_1+h} \tag{10}$$

Substituting (10) in (9)

$$F_M=1-\frac{M(H+h_1)}{h_1+h}\,\frac{\log\frac{r^1+\sqrt{(r^1)^2+(H+h_1)^2}}{r_1+\sqrt{r_1^2+(H+h_1)^2}}}{\log\frac{r^1}{r_1}}$$

$$-\frac{M(h-H)}{h_1+h}\,\frac{\log\frac{r^1+\sqrt{(r^1)^2+(h-H)^2}}{r_1+\sqrt{r_1^2+(h-H)^2}}}{\log\frac{r^1}{r_1}} \tag{11}$$

This reduces to Hayford's factor if M be put equal to 1 and H and h be put equal to zero. These are the three approximations on which Hayford's factor is obtained. If h_1 is large, it is legitimate to neglect H and h; but if h_1 were to be taken as 12 miles h and H would certainly have to be considered. This fact would increase the length of the computations since for a complete solution of the problem a different factor would be necessary for each compartment whereas, before, the same factor was used for an entire ring. Doubtless, however, devices could be employed which would facilitate the calculations.

The necessity of having to use the reduction factor given in (11) serves to make the depth and degree of completeness of compensation more open to question than ever.

SUMMARY

On the basis of Hayford's work it may be considered settled that there is some sort of isostatic compensation, but so far as Hayford's investigation has yet gone there are many possibilities as to the nature of this compensation. None of the possible distributions of compensation have been eliminated by Hayford's geodetic work; in fact, so far as the geodetic work is concerned, Hayford has shown that four different distributions of compensation are equally probable.

The present possibilities for isostatic compensation may be grouped with considerable certainty under three heads: first, there is the possibility of complete compensation at a depth in the neighborhood of 60 to 150 miles depending on the distribution of compensation; second, there is the possibility of an over-compensation at a greater depth for land areas with probably complete compensation for ocean areas; and third, there is the possibility of under-compensation at a shallow depth for land areas with complete or over-compensation for ocean areas.

SECTION III. THE THEORY OF ISOSTASY

INTRODUCTORY

In any theory of earth movements it is recognized that the earth is a failing structure in the sense that it has been and is being permanently deformed under the ultimately controlling force of gravity. It is not therefore the essential idea of the theory of isostasy that the earth as a whole is a failing structure; but the characteristic of the theory is the type of deformation which it postulates. This may not be the critical point of isostasy as it was originally conceived, or as conceived today by everyone; but it is the point which has been elaborated by the supporters of the theory and which is of first importance to the geologist; and it will therefore serve as a basis of criticism in this paper.

Type of deformation postulated in the theory of isostasy.—The controlling movements of the earth's crust are vertical movements of the various segments in response to changes of weight produced by erosion and deposition.

These vertical movements are brought about by flowage beneath the surface from areas of deposition to areas of erosion or, in general, from areas of excessive weight to areas deficient in weight.

This flowage beneath the surface is comparable in speed to the process of erosion and is started under stress-differences so small as to require that all segments of the earth of given area are essentially equal in weight.

This flowage may be accompanied by folding in the border zones of the segments.

Deformation of this kind is not restricted to the major segments, the continents and oceans, but is the type of movement which takes place between the smaller units of the continents.

Lines of criticism of isostasy.—In criticizing the theory of isostasy two main lines of argument will be followed. First, the type of deformation postulated by isostasy can not account for certain facts. Second, Hayford's geodetic results can be accounted for without supposing the type of deformation postulated by isostasy.

FACTS NOT ACCOUNTED FOR BY THE TYPE OF DEFORMATION POSTULATED IN THE THEORY OF ISOSTASY

The degree to which isostasy must be discarded depends on the importance of the phenomena which it will not explain.

The theory of isostasy as conceived in this paper does not adequately account for the folding of rocks of the earth's crust.—Folding is evidence of lateral forces of enormous magnitude. On the other hand the controlling movements of isostasy are assumed to be vertical movements. However, it has been suggested by Hayford that folding would be caused by the undertow from an area of deposition to an area of erosion:

> Horizontal compressive stresses in the material near the surface above the undertow are necessarily caused by the undertow. For the undertow necessarily tends to carry the surface along with it and so pushes this surface material against that in the region of erosion, see Fig. 2. These stresses tend to produce a crumpling, crushing and bending of the surface strata accompanied by increase of elevation of the surface. The increase of elevation of the surface so produced will tend to be greatest in the neutral region or near the edge of the region of erosion, not under the region of rapid erosion nor under the region of rapid deposition.[1]

This undertow must exist chiefly below the depth of compensation. If the earth were a perfect fluid the materials of different densities would, if not diffusible, arrange themselves in concentric shells with the heavier material toward the center. There is always a tendency for the earth to take this arrangement in the sense that the stress-differences are on an average tending in this direction. It would seem, therefore, as a general proposition, that where the material of the earth is weak, the tendency would be more toward equalization of densities laterally than toward lateral differentiation of densities such as implied by isostatic compensation. Even though the stress-differences due to lateral variations in density were not sufficient to deform the rock so as to equalize the densities, a flowage from beneath an area of deposition to an area of erosion would certainly tend to produce a distribution of density within the zone of flowage itself which would have no relation to topography. It appears, therefore, that there could be very little isostatic compensation in a zone where yielding occurs as readily as postulated by isostasy.

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[1] *Science*, February 10, 1911, p. 205.

Now, according to the theory of isostasy, compensation would be essentially complete, and if compensation is complete the depth of compensation as determined by Hayford's geodetic work would be as great as 60 miles. Hence, the undertow postulated by isostasy would exist chiefly below 60 miles. It is decidedly questionable that an undertow even much nearer to the surface than 60 miles would cause the observed folding in the upper few miles of the crust.

The theory of isostasy cannot account for the general uplift of sediments without folding.—If the isostatic compensation is complete any deposition of material should cause a sinking of the underlying segment. Isostasy could not therefore account for the fact that horizontal sedimentary rocks are found far above sea level unless a lowering of the sea level were supposed; but this possibility can generally be dismissed because the relative change in sea level is not registered in all parts of the world.[1]

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[1] In discussing isostatic adjustments (see *Science*, February 10, 1911) Hayford suggests that some uplifts are due to expansion and contraction caused by heating and cooling of sedimentation and erosion. These deformations, however, are not a distinctive assumption of the theory of isostasy at least as the theory is conceived in this paper; but were suggested to account for certain geological phenomena which the theory of isostasy could not explain.

At any rate, expansion due to heating effect of sediments is entirely inadequate to account for known uplifts. In making his estimate that the vertical expansion is

The theory of isostasy does not explain the apparently heterogeneous relation of uplift and subsidence to erosion and deposition.—Since isostasy postulates an adjustment or flowage which is comparable in speed to the process of erosion, a high area which is subject to erosion should be further uplifted as erosion progresses and should not be reduced to sea level until its deficient density is equalized by erosion of the lighter material at the surface and restoration of heavier material below. With a depth of compensation of 76 miles, the theory of isostasy would require greater continuous uplifts than are known to exist. As a matter of fact, some areas have been uplifted as erosion progressed and others have remained stationary. In some cases erosion to a peneplain has been followed by subsidence and in other cases by uplift.

ALTERNATE HYPOTHESES TO ACCOUNT FOR HAYFORD'S GEODETIC RESULTS

Erosion and deposition are assumed to be the principal cause of disturbance of the equilibrium condition of isostasy. Since deposition does not in general extend beyond the boundaries of the continental shelves, the cause and effect of the type of deformation postulated by isostasy would be confined to the continents proper. So far, then as distributions of density are to be made a proof of the theory of isostasy, the critical test is not in the density relation of the continental masses as a whole compared to the ocean basins, but in the completeness of compensation of the topographic features of the continents. Now it has been shown in the preceding section of this paper that, though there is very likely a complete compensation of the ocean defects of mass, yet it is a distinct possibility so

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one foot for every 33 feet of deposition Hayford neglects the fact that the irregularities in the isothermal surfaces near the surface of the earth flatten out with depth. However, taking Hayford's estimate and assuming that an area of deposition was covered by very shallow water and that the expansion due to heating took place all at one time, the maximum uplift above sea level could not be more than one-thirty-third of the thickness of the sediments deposited. Subsequent erosion would tend to reduce this elevation and any further elevation caused by relief from eroded material would certainly not more than equal the eroded layer. Hence, wherever the present elevation above sea level is more than one-thirty-third the thickness of the last conformable sedimentary series, some other factor than expansion due to heating effect of sedimentation must be sought to account for the uplift.

far as the geodetic evidence is concerned that the compensation of the topographic features of the continent is decidedly incomplete. The theory of isostasy can not therefore be considered as established by the geodetic work of Hayford. Furthermore, the probability that isostasy exists is lessened by the fact that an incomplete compensation can be very plausibly explained without involving the conceptions of isostasy.

The tendency of lateral compression to produce isostatic compensation.—In the folding and overthrust faulting of rocks there is abundant evidence of lateral compression. It has already been shown that this folding is probably not caused by an undertow such as isostasy supposes to be set in motion by erosion and deposition. The compression indicated by folding may be due to shrinkage of the earth; it may be due to squeezing of the continental segments by the oceanic segments; or it may be due to other causes; but whatever the cause may be, it is certain that it has produced great uplifts. Suppose that the continent is composed of portions of different densities, but that the stress-differences set up by these differences in density are not sufficient to cause a deformation of the material and a consequent uplift of the lighter masses. If this were the case, it would seem reasonable to believe that there would be a tendency for the effects of lateral compression to localize in the lighter segments since there is always a tendency for lighter segments to move up even though the stress-differences tending in this direction are not sufficient to produce an actual movement. Other things being equal, folding would probably tend to localize in sedimentary rocks since the parallel bedding planes allow slipping to take place readily. Here again there might be a tendency toward isostatic compensation since sedimentary rocks on an average are lighter than igneous rocks. There are undoubtedly other factors which determine the place of folding, but it is entirely possible that uplifts by folding are incompletely compensated.

A compensation of areas which have been uplifted without folding may be accounted for in a similar way. It is possible that lateral forces similar in magnitude to those forces which produce folding at the surface, but localized at greater depth should cause a deformation which is registered at the surface simply as a general uplift. In this case also the deformation would tend to localize where the resistance to uplift were least, in other words, in the lighter segments.

The type of deformation suggested here is perfectly distinct from that postulated by isostasy. The theory of isostasy supposes that light areas are high because the strength of the material below is not sufficient to support segments different in weight. The possibility suggested in the preceding paragraphs is that high areas are light because the great deforming forces of the earth follow the path of least resistance.

The automatic compensation of uplifts and subsidences due to expansion and contraction.—It is possible that some uplifts and subsidences are due to expansion and contraction of the underlying material. Such deformations are not a distinctive assumption of the theory of isostasy. Changes of volume may be due to changes of temperature or pressure which in turn may be due to a variety of causes. Any changes of elevation caused by expansion or contraction will be automatically compensated since the weight does not change. This would be another factor tending to produce compensation which does not involve the type of deformation postulated in isostasy.

SECTION IV. SUMMARY OF CONCLUSIONS

Isostasy is a theory of earth movements based on the assumption that the lighter portions of the earth are elevated in proportion to their defect of density because the earth is not strong enough to support segments of different weights. The principal support for the theory is the geodetic work of Hayford from which it was concluded that the excesses of mass at the surface are completely compensated for by defects of density below, said defects of density extending to a depth of something like 60 to 150 miles.

It is believed that Hayford made an error in determining the degree of completeness of compensation which invalidates his conclusions, for he assumed complete compensation in calculating the depth and then used this depth to calculate the degree of completeness. Hence, instead of the single possibility of a practically complete compensation, there are, so far as has been shown from the geodetic evidence, three groups of possibilities for isostatic compensation: first, the possibility of complete compensation at a depth in the neighborhood of 60 to 150 miles depending on the distribution of compensation; second, the possibility of an over-compensation at a greater depth for land areas with probably complete compensation for ocean areas; and third, the possibility of under-compensation at a shallower depth for land areas with complete or over-compensation for ocean areas.

Hayford's geodetic results do not, therefore, constitute a proof of the theory of isostasy.

An incomplete compensation of the topographic features of the continents can be plausibly explained without supposing the type of deformation postulated by isostasy.

There are many important phenomena which the theory of isostasy will not explain.